

Correction to: “Existence, uniqueness and comparison results for BSDEs with Lévy jumps in an extended monotonic generator setting”



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The proof of (Geiss and Steinicke (2018), Theorem 3.5) needs an extra step addressing the problem that our conditions on the generator are not sufficient to guarantee the existence of the considered optional projection:

In Definition 3.3 we defined f_n as the optional projection of

$$(\omega, t, y, z, u) \mapsto {}^{o, \mathbb{J}}f(n, \omega, t, y, z, u)$$

with respect to \mathbb{F}^n (given by $\mathcal{F}_t^n := \mathcal{F}_t \cap \mathcal{J}^n$), with parameters (y, z, u) . However, this optional projection does not always exist for generators f satisfying (A1)–(A3).

Sufficient for the existence of the optional projection of a process is boundedness or non-negativity. To guarantee the existence one can replace first $f(\omega, s, y, z, u)$ by

$$f^K(\omega, s, y, z, u) = (-K) \vee f(\omega, s, y, z, u) \wedge K$$

for some $K > 0$.

Clearly, (A1) and (A2) are satisfied for f^K . Concerning (A3), one observes that only the cases where both factors of $(y - y')(f(s, y, z, u) - f(s, y', z', u'))$ are either positive or negative are relevant. Since

$$\begin{aligned} \min \{ f(s, y, z, u) - f(s, y', z', u'), 0 \} &\leq f^K(s, y, z, u) - f^K(s, y', z', u') \\ &\leq \max \{ f(s, y, z, u) - f(s, y', z', u'), 0 \}, \end{aligned}$$

(A3) is satisfied for f^K . The above inequality implies that also (A γ) holds for f^K .

Hence in order to prove Theorem 3.5, one first starts with f^K and f'^K and gets

$$Y_t^K \leq Y_t'^K \quad \mathbb{P}\text{-a.s.}$$

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Next we will see that $\|Y_t - Y_t^K\|$ and $\|Y_t' - Y_t'^K\|$ converge to zero for $K \rightarrow \infty$, so that $Y_t \leq Y_t'$ \mathbb{P} -a.s. follows. In the proof of Proposition 4.2 it was shown that for data (ξ, f) and (ξ, f^K) it holds

$$\begin{aligned} & \sup_{t \in [0, T]} \|Y_t - Y_t^K\|^2 \\ & \leq h \left(a, b, 2\mathbb{E} \int_0^T |Y_t - Y_t^K| \left| f(t, Y_t, Z_t, U_t) - f^K(t, Y_t, Z_t, U_t) \right| dt \right). \end{aligned}$$

To see that the r.h.s. goes to zero, one can use that

$$\begin{aligned} & \mathbb{E} \int_0^T |Y_t - Y_t^K| \left| f(t, Y_t, Z_t, U_t) - f^K(t, Y_t, Z_t, U_t) \right| dt \\ & \leq \sqrt{T} \sup_{t \in [0, T]} \|Y_t - Y_t^K\| \left(\mathbb{E} \int_0^T \left| f(t, Y_t, Z_t, U_t) - f^K(t, Y_t, Z_t, U_t) \right|^2 dt \right)^{1/2}. \end{aligned}$$

The factor $\sup_{t \in [0, T]} \|Y_t - Y_t^K\|$ is bounded according to Proposition 4.1, and the integral goes to zero by monotone convergence. Since $\lim_{x \rightarrow 0} h(a, b, x) = 0$, one derives that $\lim_{K \rightarrow \infty} \|Y_t - Y_t^K\| = 0$, and in the same way it follows $\lim_{K \rightarrow \infty} \|Y_t' - Y_t'^K\| = 0$.

Moreover, Theorem 3.4 and Lemma 5.1 in Geiss and Steinicke (2018) are only valid, if f_n in Definition 3.3 exists. For the proof of Theorem 3.5 this does not cause a problem since we need these results for f_n^K only.

For more general conditions for the existence of an optional projection than non-negativity or boundedness we refer to (Dellacherie and Meyer (1982), Remarks VI.44.(f) and (He et al. 1992)).

Authors' contributions

Both authors read and approved the final manuscript.

Competing interests

The authors declare that they have no competing interests.

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